Contribution to the electrostatics of bi-isotropic materials for the case of charged ring placed near to a sphere of a bi-isotropic material

Žaklina J. Mančić, Zlata Ž. Cvetković and Saša S. Ilić

Abstract –In this paper, calculation of electric and magnetic scalar potential of charged ring in the air, which is placed near to the bi-isotropic sphere, so that its axis coincides with the radial direction of the sphere, is performed. The problem is solved using the image theorem in the bi-isotropic spherical mirror.

Keywords: electrostatics, bi-isotropic sphere, ring of charge.

I. INTRODUCTION

The circular lineic ring appears very often as an element of various systems in electromagnetics [1], such as Helmholtz coils [2], systems for generating homogeneous fields [3], or systems for space acquisition [4]. In such systems it is necessary to determine the potential, the field of the system, and if the body is placed in such system, redistribution of the field, which occurs due to the influence of that body, should be found [5].

A. Foundations of electrostatic analysis of bi-isotropic materials

In the last decades, bi-isotropic materials have been very often encountered in various problems of electromagnetics [6-15]. Bi-isoptropic materials are described using the following constitutive relations [6,7]

$$\mathbf{D} = \varepsilon\mathbf{E} + \xi\mathbf{H}, \quad \mathbf{B} = \mu\mathbf{H} + \xi\mathbf{E}. \tag{1}$$

According to Telegen, such materials consist of elements that have permanent electrical and magnetic dipoles, parallel or antiparallel with others, so that the electric field in such material simultaneously regulates both electrical and magnetic dipoles. Similarly, the magnetic field in such material regulates both electric and magnetic dipoles at the same time.

Starting from the constituent relation (1) and Maxwell's equations[2], the Laplace's equation for the electric scalar potential is obtained:

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$$\Delta\varphi = -\rho_s / \varepsilon_e, \; \varepsilon_e = \varepsilon(1 - \xi^2 / \varepsilon\mu), \tag{2}$$

as well Laplace's equation for the magnetic scalar potential of free electric charges in the bi-isotropic environment:

$$\Delta\varphi_m = \xi\rho_s / (\varepsilon_e\mu), \tag{3}$$

where $\varepsilon\mu \neq \xi^2$ [8]. When determining the electric field in such environments, firstly it is necessary to integrate the Poisson's equations for the electric and magnetic scalar potential, with respecting the existing boundary conditions, and then using the defining relations determine field components:

$$E = -\operatorname{grad}\varphi \text{ and } H = -\operatorname{grad}\varphi_m. \tag{4}$$

In this paper, a theoretical calculation of the electric and magnetic scalar potential for the case of a charged ring placed beside the bi-isotropic sphere, is presented as a contribution to the electrostatics of bi-isotropic materials. It should be noted that the axis of the ring coincides with one radial direction of the sphere.

II. The ring of charge beside the bi-isotropic sphere

Let's observe lineic ring of charge described with radius a, charged with q, which is located beside bi-isotropic sphere of radius b, placed so that the axis of the ring coincidents with the z axis of the Cartesian coordinate system, as it is shown in Fig. 1.1. The system is placed in the air/vacuum (ε_0, μ_0).

Starting from the constituent relations for bi-isotropic environments (1), outside the bi-isotropic sphere the charges produce electrical and magnetic scalar potential, that are calculated according to (2) and (3), respectively.

The solutions for the electric and magnetic scalar potential can be assumed using an expansion of the axial potential distribution into a Chebyshev series [1]:

$$\varphi = \begin{cases} \sum_{n=0}^{\infty}\left(A_n r^n + \frac{B_n}{r^{n+1}} \right) P_n(\cos\theta), \text{ for } r \le b \\ \sum_{n=0}^{\infty}\left(C_n r^n + \frac{D_n}{r^{n+1}} \right) P_n(\cos\theta), \text{for } b \le r \le \sqrt{a^2+b^2} \\ \sum_{n=0}^{\infty}\left(\left(C_n - \frac{qP_n(\cos\theta)}{4\pi\varepsilon_0 r_0^{n+1}} \right) r^n + \left(D_n + \frac{qP_n(\cos\theta) r_0^{n1}}{4\pi\varepsilon_0} \right), \frac{1}{r^{n+1}} \right) P_n(\cos\theta), \text{ for } \sqrt{a^2+b^2} \le r, \end{cases} \tag{5}$$

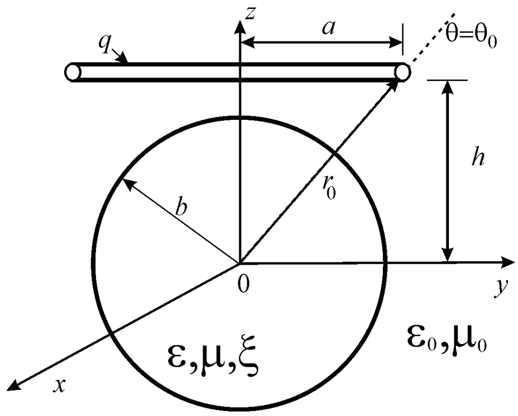

Fig. 1. Lineic ring of charge beside bi-isotropic sphere

as well as

$$\varphi_m = \begin{cases} \sum_{n=0}^{\infty}\left(A_{n1} r^n + \frac{B_{n1}}{r^{n+1}} \right) P_n(\cos\theta), r \le b \\ \sum_{n=0}^{\infty}\left(C_{n1} r^n + \frac{D_{n1}}{r^{n+1}} \right) P_n(\cos\theta), b \le r \end{cases}, \tag{6}$$

where: $A_n, B_n, C_n, D_n, A_{n_1}, B_{n_1}, C_{n1}, D_{n1}$ are the unknown constants determined using boundary conditions. Bearing in mind that the electrical and magnetic scalar potential in the center of the system ($r = 0$) must have a finite value, wherease at infinitely large distances ($r \to 0$) is equal to zero, it follows that:

$$B_n = 0\,,\ C_n = \frac{qP_n(\cos\theta)}{4\pi\varepsilon_0 r_0^{n+1}} \tag{7}$$

$$B_{n1} = 0\,,\ C_{n1} = 0\,. \tag{8}$$

Using the condition of the potential continuity on the boundary surface between two mediums (bi-isotropic sphere and air, $r = b$) as well as using the condition of the continuity of vectors $\mathbf{D}_n$ and $\mathbf{B}_n$ for $r = b$, a system of equations is obtained from which the other unknown constants are determined:

$$A_n b^n = C_n b^n + \frac{D_n}{b^{n+1}}\,, \tag{9}$$

$$D_{n1} = A_{n1} b^{2n+1}\,, \tag{10}$$

$$\varepsilon A_n n r^{n-1}\Big|_{r=b} + \xi A_{n1} n r^{n-1}\Big|_{r=b} = \varepsilon_0\left(C_n n r^{n-1} - (n+1)\frac{D_n}{r^{n+2}} \right), \tag{11}$$

$$\mu A_{n_1} n r^{n-1}\Big|_{r=b} + \xi A_n n r^{n-1}\Big|_{r=b} = -\mu_0 \frac{n+1}{r^{n+2}} D_{n1}\,. \tag{12}$$

From (7) and (9) it is obtained:

$$D_n = A_n b^{2n+1} - \frac{qP_n(\cos\theta_0)}{4\pi\varepsilon_0 r_0^{n+1}} b^{2n+1}\,. \tag{13}$$

Next, from (11) and (12):

$$n\varepsilon A_n n b^{n-1} + \xi n A_{n1} b^{n-1} = \varepsilon_0 \frac{qP_n(\cos\theta_0)}{4\pi\varepsilon_0 r_0^{n+1}} n b^{n-1} - \frac{n+1}{b^{n+2}}\varepsilon_0 D_n \tag{14}$$

and

$$\mu n A_{n1} b^{n-1} + \xi n A_n b^{n-1} = -\frac{\mu_0(n+1)}{b^{n+2}} D_{n1}\,. \tag{15}$$

Finally, it is obtained:

$$A_n = \frac{q(2n+1)P_n(\cos\theta_0)[\mu n + \mu_0(n+1)]}{4\pi r_0^{n+1}\left[[n\varepsilon + (n+1)\varepsilon_0][\mu n + \mu_0(n+1)] - \xi^2 n^2\right]} \tag{16}$$

$$A_{n1} = \frac{-\xi n q(2n+1)P_n(\cos\theta_0)}{4\pi r_0^{n+1}\left[[n\varepsilon + (n+1)\varepsilon_0][\mu n + \mu_0(n+1)] - \xi^2 n^2\right]} \tag{17}$$

$$D_n = q\frac{\left[\xi^2 n^2 + n(\varepsilon_0-\varepsilon)[\mu n+\mu_0(n+1)]\right]b^{2n+1}P_n(\cos\theta_0)}{4\pi\varepsilon_0 {r_0}^{n+1}\left[[n\varepsilon+(n+1)\varepsilon_0][\mu n+\mu_0(n+1)]-\xi^2 n^2\right]} \quad (18)$$

$$D_{n1} = \frac{-\xi n(2n+1)qb^{2n+1}P_n(\cos\theta_0)}{4\pi {r_0}^{n+1}\left[[n\varepsilon+(n+1)\varepsilon_0][\mu n+\mu_0(n+1)]-\xi^2 n^2\right]}. \quad (19)$$

In the absence of bi-isotropy, for $\xi = 0$, it becomes:

$$A_{n1} = D_{n1} = 0 \quad (20)$$

and the magnetic field does not exist, while constants A_n and D_n become equal to A_{n0} and D_{n0}, respectively, which are calculated in [1], in the case of a dielectric sphere (ε, μ):

$$A_{n0} = \frac{q(2n+1)P_n(\cos\theta_0)}{4\pi {r_0}^{n+1}[n\varepsilon+(n+1)\varepsilon_0]} \quad (21)$$

and

$$D_{n0} = q\frac{n(\varepsilon_0-\varepsilon)b^{2n+1}P_n(\cos\theta_0)}{4\pi\varepsilon_0 {r_0}^{n+1}[n\varepsilon+(n+1)\varepsilon_0]}. \quad (22)$$

Let's denote as:

$$\delta_1 = [n\varepsilon+(n+1)\varepsilon_0] \text{ and } \delta_2 = [\mu n+\mu_0(n+1)]. \quad (23)$$

Now, it is valid:

$$\frac{A_n}{A_{n0}} = \frac{1}{1-\frac{\xi^2 n^2}{\delta_1\delta_2}} = \frac{1}{1-\chi} \quad (24)$$

where

$$\chi = \frac{\xi^2 n^2}{\delta_1\delta_2} \quad (25)$$

and

$$\frac{D_n}{D_{n0}} = \frac{\left[\xi^2 n^2 + n(\varepsilon_0-\varepsilon)\delta_2\right]\delta_1}{n(\varepsilon_0-\varepsilon)\left[\delta_1\delta_2-\xi^2 n^2\right]} = \frac{1+\gamma}{1-\gamma}, \quad (26)$$

where is

$$\gamma = \frac{\xi^2 n}{(\varepsilon_0-\varepsilon)\delta_1\delta_2}. \quad (27)$$

The presence of bi-isotropy changes the values of the constants A_n and D_n and thus changes the distribution of the potential in the vicinity of the ring in the presence of a bi-isotropic sphere. In order to illustrate the dependence of the constants which apper in the expression of electric scalar potential from intensity of sphere bi-isotropy, the dependence of $\frac{A_n}{A_{n0}}$ and $\frac{D_n}{D_{n0}}$ upon $\frac{\xi^2}{\varepsilon_0\mu_0}$. Let's adopt: $n=1$, $\varepsilon=\varepsilon_r\varepsilon_0$, $\varepsilon_r=2$, $\mu=\mu_r\mu_0$, $\mu_r=1$.

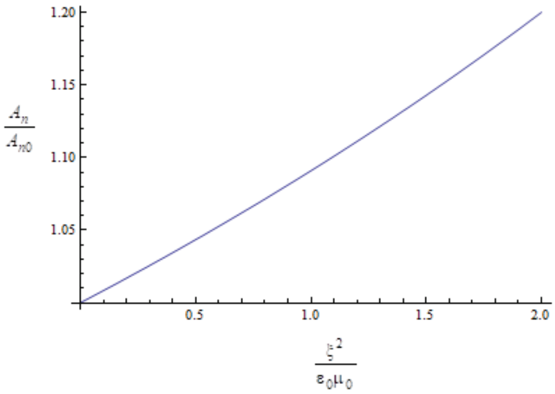

Fig. 2. Dependency $\frac{A_n}{A_{n0}}$ from $\frac{\xi^2}{\varepsilon_0\mu_0}$, for $\varepsilon = 2\varepsilon_0, \mu=\mu_0, n=1$

In Fig.2, where is shown dependancy $\frac{A_n}{A_{n0}}$ from $\frac{\xi^2}{\varepsilon_0\mu_0}$, it can be observed increasing funciton. Thereby, it has to be fullfilled $0 \le \frac{\xi^2}{\varepsilon\mu} < 1$ [8].

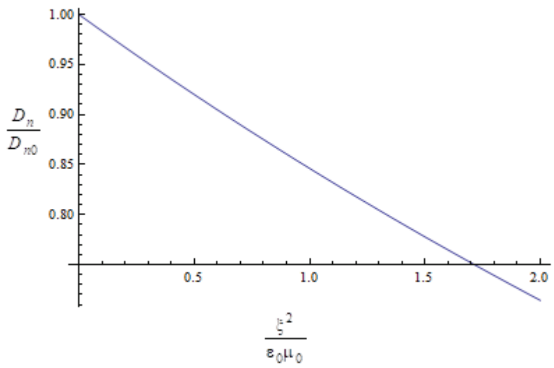

Fig. 3. Dependency $\frac{D_n}{D_{n0}}$ from $\frac{\xi^2}{\varepsilon_0\mu_0}$, for $\varepsilon = 2\varepsilon_0, \mu=\mu_0, n=1$

From Fig. 3, where the dependency of $\frac{D_n}{D_{n0}}$ upon the parameter $\frac{\xi^2}{\varepsilon_0\mu_0}$, it can be observed that this is a decreasing function that changes the sign for high values of bi-isotropy.

IV. CONCLUSION

In the paper, we have shown the results of electric and magnetic scalar potential calculation for the case when lineic ring of charge is placed near bi-isotropic sphere. It is shown that for bi-isotropic mediums, Poisson's equation can be solved, i.e. Laplace's equation in spherical coordinate system with satisfaction of boundary condition for continuity of potential and normal component of vectors **D** and **B** on the surface of bi-isotropic sphere. Bi-isotropic materials produce magnetic field when they are located in the electric field and the influence of parameter of bi-isotropy ξ is visible in intensity of the field. It is concluded that constants in expressions for electric and magnetic scalar potential, depend on the parameter which describes bi-istropic medium, ξ . For the case $\xi = 0$, the expressions for electric scalar potential become equal to those in [1] and magnetic field does not exist.

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